OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The point on the x-axis which is equidistant from (–4, 0) and (10, 0) is:

(a) (7, 0) (b) (5, 0)

(c) (0, 0) (d) (3, 0) [Diksha]

Ans. *(d) (3, 0)*

Explanation: Since, both the given points are on the x-axis, the mid-point $\left(\frac{-4+10}{2}, \frac{0+0}{2}\right)$, *i.e.*, (3, 0) lies on x-axis and is equidistant from (–4, 0) and (10, 0).

2. The distance between the points (m, –n) and (–m, n) is:

(a) $\sqrt{m^2+n^2}$ (b) $m+n$

(c) $2\sqrt{m^2+n^2}$ (d) $\sqrt{2m^2+n^2}$

[CBSE 2011]

Ans. *(c)* $2\sqrt{m^2+n^2}$

Explanation: Distance between $(m, -n)$ and $(-m, n)$

$$= \sqrt{(-m-m)^2+(n+n)^2}$$

(By distance formula)

$$= \sqrt{4m^2+4n^2}$$

$$= 2\sqrt{m^2+n^2}$$

3. If the vertices of a parallelogram PQRS taken in order are P(3, 4), Q(–2, 3) and R(–3, –2), then the coordinates of its fourth vertex S are:

(a) (–2, –1) (b) (–2, –3)

(c) (2, –1) (d) (1, 2)

[CBSE SQP Std. 2022]

Ans. *(c) (2, –1)*

[CBSE Marking Scheme SQP Std. 2022]

Explanation: Since, the diagonals of a parallelogram bisect each other.

Let the coordinate of S is (x, y)

∴ Coordinates of the mid point of PR = coordinates of the mid point of QS

$$\Rightarrow \left(\frac{3+(-3)}{2}, \frac{4-2}{2}\right) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$$

$$(0, 1) = \left(\frac{x-2}{2}, \frac{3+y}{2}\right)$$

$$\frac{x-2}{2} = 0 \text{ and } \frac{3+y}{2} = 1$$

$\Rightarrow$ $x - 2 = 0$

$x = 2$

and $3 + y = 2$

$y = -1$

Hence, the fourth vertex of the parallelogram is (2, –1).

4. The point which divides the line segment joining the points (7, –6) and (3, 4) in the ratio 1 : 2 internally, lies in the:

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

[CBSE 2011]

5. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$, is:

(a) a^2+b^2 (b) a^2-b^2

(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

[CBSE 2011]

Ans. *(c)* $\sqrt{a^2+b^2}$

Explanation: Distance between the given points

$$= \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$$

$$= \sqrt{\begin{aligned}&a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta \\ &+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta\end{aligned}}$$

$$= \sqrt{a^2(\cos^2\theta+\sin^2\theta) + b^2(\sin^2\theta+\cos^2\theta)}$$

$$= \sqrt{a^2+b^2} \quad [\because \cos^2\theta + \sin^2\theta = 1]$$

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is:

(a) 5 units (b) 12 units

(c) 11 units (d) $(7+\sqrt{5})$ units

[CBSE SQP Basic 2022]

Ans. *(b) 12 units*

[CBSE Marking Scheme SQP Basic 2022]

Explanation: Let A(0, 4), O(0, 0) and B(3, 0) be the vertices of ΔAOB.

Using distance formula, we get

$$OA = \sqrt{(0-0)^2+(4-0)^2} = \sqrt{16} = 4 \text{ units}$$

$$OB = \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3 \text{ units}$$

$$AB = \sqrt{(3-0)^2+(0-4)^2} = \sqrt{9+16} = \sqrt{25}$$

$= 5$ units

∴ Perimeter of ΔAOB = OA + OB + AB

$= 4 + 3 + 5$

$= 12$ units

Thus, the required perimeter of the triangle is 12 units.

7. The point which lies on the perpendicular bisector of the line segment joining points A (–2, –5) and B (2, 5) is:

(a) (0, 0) (b) (0, –1)

(c) (–1, 0) (d) (1, 0)

[NCERT Exemplar]

8. If the point P(k, 0) divides the line segment joining the points A(2, –2) and B(–7, 4) in the ratio 1 : 2, then the value of k is:

(a) 1 (b) 2

(c) –2 (d) –1 [CBSE 2011]

Ans. *(d) –1*

Explanation: Here,

$$\underset{A(2,-2)}{\overset{1}{\overset{(k,0)}{\overset{P}{\overset{2}{\underset{B(-7,4)}{}}}}}}$$

Here, $P(k, 0) = \left(\frac{1\times(-7)+2\times2}{1+2}, \frac{1\times4+2\times(-2)}{1+2}\right)$

[By Section Formula]

$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$ *i.e.*, $(-1, 0)$

$\Rightarrow \quad k = -1$

9. The distance of the point P(–3, –4) from the *x*-axis (in units) is:

(a) 3 **(b) –3**

(c) 4 **(d) 5**

[CBSE 2019]

Ans. *(c) 4*

Explanation: Perpendicular distance of a point from *x*-axis = |*y*-coordinate of that point|

$\therefore$ For point (–3, –4),

Required distance = |–4| = 4 units

10. If $A\left(\frac{m}{3}, 5\right)$ is the mid-point of the line segment joining the points Q(–6, 7) and R(–2, 3), then the value of *m* is:

(a) –12 **(b) –4**

(c) 12 **(d) –6** **[CBSE 2019]**

Ans. *(a) –12*

Explanation: $A\left(\frac{m}{3}, 5\right)$ is the mid-point of Q(–6, 7) and R(–2, 3).

$\therefore \quad \frac{m}{3} = \frac{-6-2}{2}$

$\Rightarrow \quad \frac{m}{3} = -4$

$\Rightarrow \quad m = -12$

11. The perimeter of a triangle ABC with vertices A(0, 4), B(0, 0) and C(3, 0) is:

(a) 5 units **(b) 11 units**

(c) 12 units **(d) $(7+\sqrt{5})$ units**

[CBSE 2014, NCERT Exemplar]

Ans. *(c) 12 units*

Explanation: Perimeter of triangle ABC

= AB + BC + CA

$= \sqrt{(0-0)^2+(4-0)^2} + \sqrt{(3-0)^2+(0-0)^2} + \sqrt{(3-0)^2+(0-4)^2}$

$= \sqrt{16} + \sqrt{9} + \sqrt{25}$

$= 4 + 3 + 5$

$= 12$ units

12. In which ratio the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4)?

(a) 1 : 5 **(b) 5 : 1**

(c) 1 : 1 **(d) 1 : 2**

[CBSE SQP Basic 2022]

Ans. *(b) 5 : 1*

[CBSE Marking Scheme SQP Basic 2022]

Explanation: Using the section formula, if a point (x, y) divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$, then

$$(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Let the point on *y*-axis be P(0, *y*) and

AP : PB = *k* : 1

Therefore, using section formula:

$$\frac{5\times1+(-1)\times k}{k+1} = 0$$

$$5 - k = 0$$

$\Rightarrow \quad k = 5$

Hence, required ratio is 5 : 1.

Caution

In such problems be clear about the ratio, in which a particular point divides the given line, otherwise the points we get would be wrong.

13. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q(–6, 5) and R(–2, 3), then the value of *a* is:

(a) –4 **(b) –12**

(c) 12 **(d) –6** **[CBSE 2010]**

14. The perpendicular bisector of the line segment joining the points A(1, 5) and B(4, 6) cuts the *y*-axis at:

(a) (0, 13) **(b) (0, –13)**

(c) (0, 12) **(d) (13, 0)**

[NCERT Exemplar]

15. The relation *x* and *y* and so that the point (*x*, *y*) is equidistant from the points (–4, –4) and (–2, 4) is:

(a) $x - 4y + 3 = 0$ **(b) $x - 4y - 3 = 0$**

(c) $x + 4y - 3 = 0$ **(d) $x + 4y + 3 = 0$**

[Delhi Gov. SQP 2022]

Ans. *(d)* $x + 4y + 3 = 0$

Explanation: Let the given points be written as, A(–4, –4), B(–2, 4) and P(*x*, *y*)

It is told that point P is equidistant from points A and B.

It can be mathematically denoted as PA = PB.

The distance PA is,

$$PA = \sqrt{(-4-x)^2+(-4-y)^2}$$

The distance PB is,

$$PB = \sqrt{(-2-x)^2+(4-y)^2}$$

Equating both of these we get

$$\sqrt{(-4-x)^2+(-4-y)^2} = \sqrt{(-2-x)^2+(4-y)^2}$$

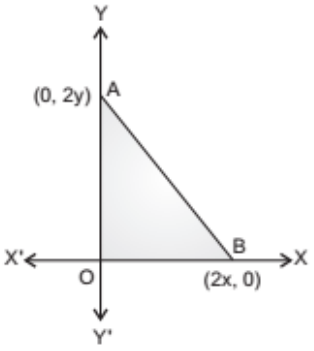
$$\Rightarrow (-4-x)^2 + (-4-y)^2 = (-2-x)^2 + (4-y)^2$$

$$\Rightarrow x^2 + y^2 + 8x + 8y + 32 = x^2 + y^2 + 4x - 8y + 20$$

$$\Rightarrow 4x + 16y + 12 = 0$$

$$\Rightarrow x + 4y + 3 = 0$$

16. The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is:



(a) (x, y) **(b)** (y, x)

(c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ **(d)** $\left(\frac{y}{2}, \frac{x}{2}\right)$

[NCERT Exemplar]

Ans. *(a)* (x, y)

Explanation: Let P(*h*, *k*) be a point equidistant from the three vertices of ΔAOB: A(0, 2*y*), B(2*x*, 0) and O(0, 0).

Then, $PO = PA = PB$

$$\Rightarrow (PO)^2 = (PA)^2 = (PB)^2 \quad ...(i)$$

By distance formula,

$$(PO)^2 = \left(\sqrt{(h-0)^2+(k-0)^2}\right)^2 = h^2 + k^2$$

$$(PA)^2 = \left(\sqrt{(h-0)^2+(k-2y)^2}\right)^2 = h^2 + (k-2y)^2$$

$$(PB)^2 = \left(\sqrt{(h-2x)^2+(k)^2}\right)^2 = (h-2x)^2 + (k)^2$$

Putting value in eqn (i), we get

$$h^2 + k^2 = h^2 + (k-2y)^2$$

$$\Rightarrow h^2 + k^2 = h^2 + k^2 + 4y^2 - 4ky$$

$$\Rightarrow 4y^2 - 4ky = 0$$

$$\Rightarrow 4y(y-k) = 0$$

$$\Rightarrow y = k \quad [\because y \neq 0]$$

Also, $h^2 + k^2 = (h-2x)^2 + k^2$

$$\Rightarrow h^2 + k^2 = h^2 + 4x^2 - 4hx + k^2$$

$$\Rightarrow 4x^2 - 4hx = 0$$

$$\Rightarrow 4x(x-h) = 0$$

$$\Rightarrow x = h \quad [\because x \neq 0]$$

$\therefore$ Required point = $(h, k) = (x, y)$.

Alternative Method:

In any right triangle, the mid-point of the hypotenuse is equidistant from the three vertices.

Hence, the coordinates of the required point are $\left(\frac{2x+0}{2}, \frac{0+2y}{2}\right)$ *i.e.* (x, y)

Concept Applied

- *Step 1. Consider new point to be P (h, k).*
- *Step 2. Determine PO, PA and PB using distance formula and equate them.*
- *Step 3. Solve two terms at a time to get required point.*

17. The relation between x and y so that the point (x,y) is equidistant from the points (-4,-4) and (-2,4) is:

(a) 2 **(b)** 5

(c) 3 **(d)** 8

[Delhi Gov. SQP 2022]

Ans. *(a)* 2

Explanation: Since A and B lie on the circle having centre O.

Therefore, $OA = OB$

$$\sqrt{(4-2)^2+(3-3)^2} = \sqrt{(x-2)^2+(5-3)^2}$$

$$2 = \sqrt{(x-2)^2+4}$$

$$(x-2)^2 + 4 = 4$$

$$(x-2)^2 = 0$$

$$x - 2 = 0$$

$$x = 2$$

18. Three vertices of a parallelogram ABCD are A(1, 4), B(–2, 3) and C(5, 8). The ordinate of the fourth vertex D is:

(a) 8 **(b)** 9

(c) 7 **(d)** 6

[CBSE Term-1 Std. 2021]

Ans. *(b) 9*

Explanation: Let the coordinates of D be (x, y). Since, ABCD is a parallelogram, so diagonals AC and BD bisect each other at a point.

$\therefore$ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{1+5}{2}, \frac{4+8}{2}\right) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$$

$$\Rightarrow x - 2 = 6;\ y + 3 = 12$$

$$\Rightarrow x = 8;\ y = 9$$

Hence, the ordinate of D is 9.

19. **If $A\left(3, \sqrt{3}\right)$, B(0, 0) and C(3, k) are the three vertices of an equilateral triangle ABC, then the value of k is:**

(a) 2 **(b) –2**

(c) $-\sqrt{3}$ **(d) $-\sqrt{2}$**

[CBSE Term-1 Std. 2021]

20. **A circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$. The point which does not lie in the interior of the circle is:**

(a) $\left(\frac{-3}{4}, 1\right)$ **(b) $\left(2, \frac{7}{3}\right)$**

(c) $\left(5, \frac{-1}{2}\right)$ **(d) $\left(-6, \frac{5}{2}\right)$** **[Diksha]**

21. **The vertices of a parallelogram in order are A(1, 2), B(4, y), C(x, 6) and D(3, 5). Then (x, y) is:**

(a) (6, 3) **(b) (3, 6)**

(c) (5, 6) **(d) (1, 4)**

[CBSE Term-1 SQP 2021]

Ans. *(a) (6, 3)*

Since, ABCD is a parallelogram, diagonals AC and BD bisect each other.

$\therefore$ mid-point of AC = mid point of BD.

$$\left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$$

Comparing the co-ordinates, we get

$$\frac{x+1}{2} = \frac{3+4}{2}$$

$$x = 6$$

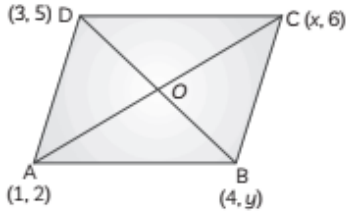
Similarly, $\frac{5+y}{2} = \frac{6+2}{2}$.

So $y = 3$

$\therefore$ $(x, y) = (6, 3)$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: We know, diagonals of a parallelogram bisect each other.



$\therefore$ Mid-point of diagonal AC = Mid-point of BD

$$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \frac{1+x}{2} = \frac{4+3}{2};\ \frac{2+6}{2} = \frac{y+5}{2}$$

$$\Rightarrow x + 1 = 7\ ;\ 8 = y + 5$$

$$\Rightarrow x = 6\ ;\ y = 3$$

$$\Rightarrow (x, y) = (6, 3)$$

22. **Point P divides the line segment joining R(–1, 3) and S(9, 8) in ratio k : 1. If P lies on the line $x - y + 2 = 0$, then value of k is:**

(a) $\frac{2}{3}$ **(b) $\frac{1}{2}$**

(c) $\frac{1}{3}$ **(d) $\frac{1}{4}$**

[CBSE Term-1 SQP 2021]

23. **The ratio in which the line $3x + y - 9 = 0$ divides the line segment joining the points (1, 3) and (2, 7) is:**

(a) 3 : 2 (b) 2 : 3

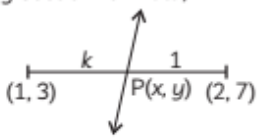
(c) 3 : 4 (d) 4 : 3

[CBSE Term-1 Std. 2021]

Ans. *(c) 3 : 4*

Explanation: Let P(x, y) be the point on the line $3x + y - 9 = 0$ which divide the line joining (1, 3) and (2, 7) in the ratio k : 1.

So, using section formula,



$$3x + y - 9 = 0$$

$$P(x, y) = \left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$$

Now, point P lies on the line $3x + y - 9 = 0$, so its coordinates must satisfy the line.

$$\therefore \quad 3\left(\frac{2k+1}{k+1}\right) + \left(\frac{7k+3}{k+1}\right) - 9 = 0$$

$\Rightarrow 3(2k+1)+(7k+3)-9(k+1)=0$

$\Rightarrow 6k+3+7k+3-9k-9=0$

$\Rightarrow 4k-3=0$

$\Rightarrow k=\frac{3}{4}$

$\therefore$ Ratio $= k:1$

$=\frac{3}{4}:1=3:4$

24. Points A(–1, y) and B(5, 7) lie on a circle with centre O(2, –3y). The values of y are :

(a) 1, –7 **(b) –1, 7**

(c) 2, 7 **(d) –2, –7**

[CBSE Term-1 Std. 2021]

Ans. *(b) –1, 7*

Explanation: Since, points A and B lie on a circle with centre O

$\therefore$ OA = OB [Radii]

or, $(OA)^2=(OB)^2$

$\Rightarrow (2+1)^2+(-3y-y)^2=(2-5)^2+(-3y-7)^2$

[Using distance formula]

$\Rightarrow (3)^2+(-4y)^2=(-3)^2+(-3y-7)^2$

$\Rightarrow 9+16y^2=9+9y^2+49+42y$

$\Rightarrow 7y^2-42y-49=0$

$\Rightarrow y^2-6y-7=0$

$\Rightarrow y^2-7y+y-7=0$

$\Rightarrow (y-7)(y+1)=0$

$\Rightarrow y=7,-1$

25. The point P which divides the line segment joining the points A(2, –5) and B(5, 2) in the ratio 2 : 3 lies in the quadrant:

(a) I **(b) II**

(c) III **(d) IV** **[CBSE 2011]**

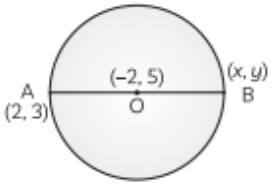
26. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre (–2, 5) then the coordinates of the other end of the diameter are:

(a) (–6, 7) **(b) (6, 7)**

(c) (6, –7) **(d) (–6, –7)**

Ans. *(a) (–6, 7)*

Explanation : Let O be the centre of the circle with diameter AB, where A = (2, 3) and B = (x, y) (say).



Then, O is the mid-point of AB.

$\Rightarrow (-2,5)=\left(\frac{2+x}{2},\frac{3+y}{2}\right)$

$\therefore \frac{2+x}{2}=-2$

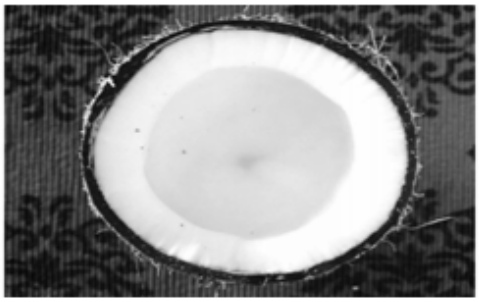
and $\frac{3+y}{2}=5$

$\Rightarrow x=-6$

and $y=7$

Hence, coordinates of the other end of the diameter are (–6, 7).

27. Parul was helping her mother in the kitchen in making coconut chutney to be eaten with dosa. Her mother had already broken the coconut and asked Parul to scrap the coconut using a scraper. Parul looked quite amazed at the near perfect circle made by the coconut shell and wondered how the coordinates of the centre of the coconut can be found with the knowledge of coordinate geometry.



The centre of a circular shaped coconut whose end points of a diameter are (–6, 3) and (6, 4), are:

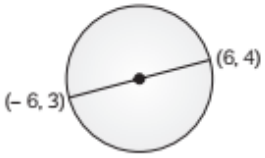
(a) (8, –1) **(b) (4, 7)**

(c) $\left(0,\frac{7}{2}\right)$ **(d) $\left(4,\frac{7}{2}\right)$**

[Mod. CBSE 2011]

Ans. *(c)* $\left(0,\frac{7}{2}\right)$

Explanation:



Since, the centre of a circle is the mid-point of its diameter, the centre is $\left(\frac{-6+6}{2},\frac{3+4}{2}\right)$, *i.e.*, $\left(0,\frac{7}{2}\right)$

28. **One day Abhimanyu, a budding interior decorator, went to his friends' house to attend a party. He looked at the beautiful wooden shelf, shaped like a parallelogram and jokingly remarked that he can use the knowledge of coordinate geometry and find the coordinates of the vertices of the parallelogram.**

The fourth vertex D of the parallelogram ABCD whose three vertices are A (–2, 3), B(6, 7) and C (8, 3), is:

(a) (0, 1) **(b) (0, –1)**
(c) (–1, 0) **(d) (1, 0)**

[Mod. CBSE 2010]

29. **While on a road trip with his friends, the wheel of Amar's vehicle got stuck in ice. With no help in sight, they managed to use a makeshift lever to raise the wheel and were safe.**

If the point P(2, 1) lies on the line segment joining the points A(4, 2) and B(8, 4), on the makeshift lever then:

(a) $AP = \frac{1}{3}AB$ **(b)** $AP = PB$

(c) $PB = \frac{1}{3}AB$ **(d)** $AP = \frac{1}{2}AB$

[Mod. NCERT]

Ans. *(d)* $AP = \frac{1}{2}AB$

Explanation: It is given that P(2, 1) lies on the line segment joining the points A(4, 2) and B(8, 4).

A(4, 2) P (2, 1) B (8, 4)

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\therefore$ Distance between A(4, 2) and P(2, 1)

$$AP = \sqrt{(2-4)^2 + (1-2)^2}$$

$$= \sqrt{(-2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

Distance between P(2, 1) and B(8, 4)

$$PB = \sqrt{(8-2)^2 + (4-1)^2}$$

$$= \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

Distance between A(4, 2) and B(8, 4)

$$AB = \sqrt{(8-4)^2 + (4-2)^2}$$

$$= \sqrt{(4)^2 + (2)^2} = \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5}$$

$\therefore AB = 2\sqrt{5} = 2(AP)$

$\Rightarrow AP = \frac{AB}{2}$.

Caution

Students should remember that section formula can be used to evaluate the ratio between the sides AP, PB and AB.

30. **Sam and his friend Alex went to a park to play. They found the see saw in the park and thought of giving it a ride. Prateek who was watching the two kids enjoying their see saw ride was trying to figure out how his knowledge of coordinate geometry will help him in finding the mid-point of the see-saw.**

If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and $a - 2b = 18$ in the see-saw, the value of k is:

(a) 30 **(b) 22**
(c) 4 **(d) 40**

[Mod. CBSE 2020]

Ans. *(b) 22*

Explanation: Mid-point of AB

$$= \left(\frac{10+k}{2}, \frac{-6+4}{2}\right) \text{ i.e., } \left(5+\frac{k}{2}, -1\right).$$

28. **One day Abhimanyu, a budding interior decorator, went to his friends' house to attend a party. He looked at the beautiful wooden shelf, shaped like a parallelogram and jokingly remarked that he can use the knowledge of coordinate geometry and find the coordinates of the vertices of the parallelogram.**

The fourth vertex D of the parallelogram ABCD whose three vertices are A (–2, 3), B(6, 7) and C (8, 3), is:

(a) (0, 1) **(b) (0, –1)**
(c) (–1, 0) **(d) (1, 0)**

[Mod. CBSE 2010]

29. **While on a road trip with his friends, the wheel of Amar's vehicle got stuck in ice. With no help in sight, they managed to use a makeshift lever to raise the wheel and were safe.**

If the point P(2, 1) lies on the line segment joining the points A(4, 2) and B(8, 4), on the makeshift lever then:

(a) $AP = \frac{1}{3}AB$ **(b)** $AP = PB$

(c) $PB = \frac{1}{3}AB$ **(d)** $AP = \frac{1}{2}AB$

[Mod. NCERT]

Ans. *(d)* $AP = \frac{1}{2}AB$

Explanation: It is given that P(2, 1) lies on the line segment joining the points A(4, 2) and B(8, 4).

A(4, 2) P (2, 1) B (8, 4)

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\therefore$ Distance between A(4, 2) and P(2, 1)

$$AP = \sqrt{(2-4)^2 + (1-2)^2}$$

$$= \sqrt{(-2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

Distance between P(2, 1) and B(8, 4)

$$PB = \sqrt{(8-2)^2 + (4-1)^2}$$

$$= \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

Distance between A(4, 2) and B(8, 4)

$$AB = \sqrt{(8-4)^2 + (4-2)^2}$$

$$= \sqrt{(4)^2 + (2)^2} = \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5}$$

$\therefore AB = 2\sqrt{5} = 2(AP)$

$\Rightarrow AP = \frac{AB}{2}$.

Caution

Students should remember that section formula can be used to evaluate the ratio between the sides AP, PB and AB.

30. **Sam and his friend Alex went to a park to play. They found the see saw in the park and thought of giving it a ride. Prateek who was watching the two kids enjoying their see saw ride was trying to figure out how his knowledge of coordinate geometry will help him in finding the mid-point of the see-saw.**

If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and $a - 2b = 18$ in the see-saw, the value of k is:

(a) 30 **(b) 22**
(c) 4 **(d) 40**

[Mod. CBSE 2020]

Ans. *(b) 22*

Explanation: Mid-point of AB

$$= \left(\frac{10+k}{2}, \frac{-6+4}{2}\right) \text{ i.e., } \left(5+\frac{k}{2}, -1\right).$$

Since it is (a, b), we have

$$b = -1, 5 + \frac{k}{2} = a$$

From the given relation $a - 2b = 18$, we have

$$\Rightarrow 5 + \frac{k}{2} - 2(-1) = 18$$

$$\Rightarrow \frac{k}{2} = 11$$

$$\therefore k = 22$$

Fill in the Blanks

31. **AOBC is a rectangle whose three vertices are A(0, –3), O(0, 0) and B(4, 0). The length of its diagonal is ..... . [CBSE 2011]**

32. **The centroid of the triangle whose vertices are (4, –8), (–9, 7) and (8, 13), is ..... .**

Ans. (1, 4)

Explanation: Centroid of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by,

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$\therefore$ Required centroid

$$= \left(\frac{4 - 9 + 8}{3}, \frac{-8 + 7 + 13}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{12}{3}\right) = (1, 4)$$

33. **The ratio in which x-axis divides the line segment joining the points (2, 3) and (4, – 8), is ..... .**

Ans. 3 : 8

Explanation: Let $(x, 0)$ be the point on x-axis which divides the line segment joining (2, 3) and (4, –8).

Let the ratio in which the line segment is divided be $k : 1$.

(2, 3) — k — $(x, 0)$ — 1 — (4, – 8)

$\therefore$ Using section formula,

$$(x, 0) = \left(\frac{4k + 2}{k + 1}, \frac{-8k + 3}{k + 1}\right)$$

$$0 = \frac{-8k + 3}{k + 1}$$

$$\Rightarrow 8k = 3$$

$$\Rightarrow k = \frac{3}{8}$$

$$\therefore \text{Ratio} = k : 1 = \frac{3}{8} : 1 = 3 : 8$$

34. **The mid-point of the line segment AB is (4, 0). If the coordinates of point A are (3, –2), then coordinates of point B are ..... .**

35. **Distance of a point (–24, 7) from the origin (in units) is ..... .**

Ans. 25

Explanation: Distance between (–24, 7) and (0, 0) is,

$$d = \sqrt{(-24 - 0)^2 + (7 - 0)^2}$$

$$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$$

$$= \sqrt{625}$$

$$= 25 \text{ units}$$

36. **If P(–1, 1) is the mid-point of the line segment joining the points A(–3, b) and B(1, b + 4), then b is .....**

Ans. –1

Explanation: Given : A(–3, b) and B(1, b + 4)

Also, mid-point of AB = (–1, 1)

$$\therefore (-1, 1) = \left(\frac{-3 + 1}{2}, \frac{b + (b + 4)}{2}\right)$$

$$\Rightarrow 1 = \frac{2b + 4}{2}$$

$$\Rightarrow 2b + 4 = 2$$

$$\Rightarrow 2b = -2$$

$$\Rightarrow b = -1$$

37. **$\sqrt{2}$ times the distance between (0, 5) and (– 5, 0) is ..... . [CBSE 2020]**

Ans. 10

Explanation: Distance between (0, 5) and (– 5, 0)

$$= \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25 + 25}$$

$$= 5\sqrt{2}$$

Now, $\sqrt{2} \times \text{distance} = \sqrt{2} \times 5\sqrt{2} = 10$

Thus, $\sqrt{2}$ times the distance between (0, 5) and (–5, 0) is 10.

38. **The distance of the point (–3, 4) from y-axis is ..... . [CBSE 2020]**

39. **The distance between the points $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$ is ..... . [CBSE 2020]**

Ans. 2 units

Explanation: Distance between the points $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$

$$= \sqrt{\left(\frac{2}{5}+\frac{8}{5}\right)^2+(2-2)^2}$$

$$= \sqrt{(2)^2+0} = \sqrt{4}$$

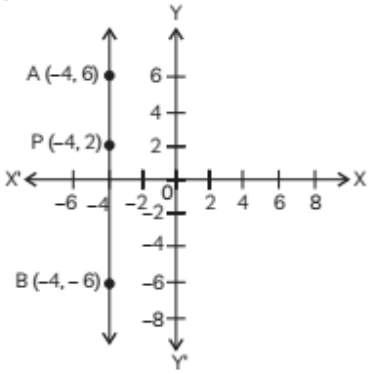
$$= 2$$

True/False

40. Point P (–4, 2) lies on the line segment joining the points A (–4, 6) and B (–4, –6). [NCERT]

Ans. True.

Explanation:



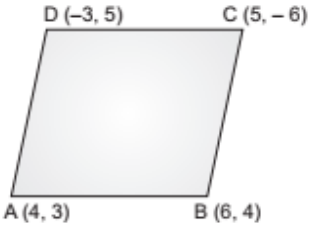
Thus, the point P (-4, 2) lies on the line segment joining the points A (-4, 6) and B(-4, -6).

41. Points A(4, 3), B(6, 4), C(5, –6) and D(–3, 5) are the vertices of a parallelogram.

[CBSE 2012, NCERT Exemplar]

Ans. False.

Explanation: We know that opposite sides of a parallelogram are equal in length.



Also, we know that, distance between the points (x_1, y_1) and (x_2, y_2), is

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

∴ Distance between A(4, 3) and B(6, 4)

$$AB = \sqrt{(6-4)^2+(4-3)^2}$$

$$= \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$$

Distance between B(6, 4) and C(5, –6)

$$BC = \sqrt{(5-6)^2+(-6-4)^2}$$

$$= \sqrt{1^2+(-10)^2} = \sqrt{1+100} = \sqrt{101}$$

Distance between C(5, –6) and D(–3, 5)

$$CD = \sqrt{(5+3)^2+(-6-5)^2}$$

$$= \sqrt{(8)^2+(-11)^2} = \sqrt{64+121}$$

$$= \sqrt{185}$$

Distance between D(–3, 5) and A(4, 3)

$$DA = \sqrt{(4+3)^2+(3-5)^2} = \sqrt{7^2+(-2)^2}$$

$$= \sqrt{49+4} = \sqrt{53}$$

As AB ≠ CD and BC ≠ DA,

Hence, the given points are not vertices of a parallelogram.

42. Point P(5, –3) is one of the two points of trisection of the line segment joining points A(7, –2) and B(1, –5).

[CBSE 2012, NCERT Exemplar]

Ans. True.

Explanation: Let point P(5, –3) divide the line segment joining the points A(7, –2) and B(1, –5) in the ratio $m : 1$ internally.

Using section formula,

Coordinates of point P will be

$$\left[\frac{m(1)+1(7)}{m+1}, \frac{m(-5)+1(-2)}{m+1}\right] = \left[\frac{m+7}{m+1}, \frac{-5m-2}{m+1}\right]$$

According to the question,

$$(5, -3) = \left(\frac{m+7}{m+1}, \frac{-5m-2}{m+1}\right)$$

$$\Rightarrow \quad 5 = \frac{m+7}{m+1}$$

and $$-3 = \frac{-5m-2}{m+1}$$

$$\Rightarrow \quad 5(m+1) = (m+7)$$

and $$3(m+1) = 5m+2$$

$$\Rightarrow \quad 5m+5-m-7 = 0$$

and $$3m+3-5m-2 = 0$$

$$\Rightarrow \quad 4m-2 = 0$$

and $$-2m+1 = 0$$

$$\Rightarrow \quad m = \frac{1}{2}$$

Hence, point P divides the line segment AB in the ratio 1 : 2.

Thus, point P is one of the points of trisection of AB.

43. Point P(–2, 4) lies on a circle of radius 6 and centre C(3, 5).

[CBSE 2014, 13, NCERT Exemplar]

Ans. False.

Explanation:

$$PC = \sqrt{(-2-3)^2+(4-5)^2} = \sqrt{(-5)^2+(-1)^2}$$

$$= \sqrt{25+1} = \sqrt{26}$$

Clearly, PC ≠ radius *i.e.*, 6

Hence, point P (–2, 4) does not lie on the circle with centre C (3, 5) and radius 6.

44. The points A(–1, –2), B(4, 3), C(2, 5) and D(–3, 0) in that order form a rectangle.

[CBSE 2013, NCERT Exemplar]

Assertion Reason

Direction for questions 45 to 49: In question number 45 to 49, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

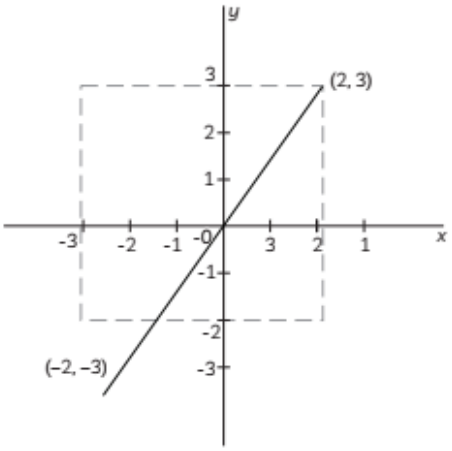
45. Assertion (A): The origin is the only point equidistant from (2, 3) and (-2, -3).

Reason (R): The origin is the midpoint of the line joining (2, 3) and (-2, -3).

[CBSE Question Bank 2023]

Ans. *(d) Assertion (A) is false but reason (R) is true.*

Explanation:



From the graph we can see, there are many equidistant points from (2, 3) and (-2, -3)

Now, by midpoint formula.

$$\left(\frac{x_1+y_1}{2}\right), \left(\frac{x_2+y_2}{2}\right) = \left(\frac{2-2}{2}\right), \left(\frac{3-3}{2}\right)$$

$$= (0, 0)$$

Thus, origin is the midpoint of the line joining (2, 3) and (-2, -3).

Hence, assertion is false but reason is true.

46. Assertion (A): If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D(3, 5) and E(–3, –3) respectively, then BC = 20 units

Reason (R): The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

[CBSE SQP Std. 2022]

47. Assertion (A):The point (0, 4) lies on *y*-axis.

Reason (R): The *x*-coordinate on the point on *y*-axis is zero.

48. Assertion (A): The value of *y* is 6, for which the distance between the points P(2, –3) and Q(10, *y*) is 10.

Reason (R): Distance between two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Ans. *(d) Assertion (A) is false but reason (R) is true.*

Explanation: We know that the distance between two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

so, distance between P(2, –3) and Q(10, *y*) is 10. then,

$$\Rightarrow \quad 10 = \sqrt{[(10-2)^2+(y+3)^2]}$$

Squaring both sides,

$$\Rightarrow \quad 10^2 = 8^2 + (y+3)^2$$

$$\Rightarrow \quad 100 - 64 = (y+3)^2$$

$$\Rightarrow \quad 36 = (y+3)^2$$

$$\Rightarrow \quad (\pm 6)^2 = (y+3)^2$$

$$\Rightarrow \quad y + 3 = \pm 6$$

therefore,

$$\Rightarrow \quad y = 6 - 3 = 3$$

$$\Rightarrow \quad y = -6 - 3 = (-9)$$

Hence, assertion is false but the reason is true.

49. **Assertion (A):** **Mid-point of a line segment divides line in the ratio 1 : 1.**
Reason (R): **The ratio in which the point (–3, *k*) divides the line segment joining the points (–5, 4) and (–2, 3) is 1 : 2.**

Ans. *(c) Assertion (A) is true but reason (R) is false.*

Explanation: We know that, the mid-point of a line segment divides line in the ratio 1 : 1

Let (–3, *k*) divides the line segment joining the points (–5, 4) and (–2, 3) in the ratio $m : n$,

then $$-3 = \frac{[(-2)m + (-5)n]}{(m + n)}$$

$$-3(m + n) = -2m - 5n$$
$$-3m - 3n = -2m - 5n$$
$$-3m + 2m = -5n + 3n$$
$$-m = -2n$$
$$m = 2n$$
$$m : n = 2 : 1$$

Hence, assertion is true but the reason is false.

CASE BASED Questions (CBQs)

[4 & 5 marks]

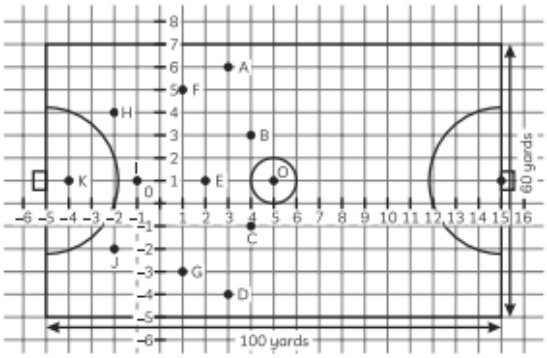
Read the following passages and answer the questions that follow:

50. *A hockey field is the playing surface for the game of hockey. Historically, the game was played on natural turf (grass) but nowadays it is predominantly played on an artificial turf.It is rectangular in shape - 100 yards by 60 yards. Goals consist of two upright posts placed equidistant from the centre of the backline, joined at the top by a horizontal crossbar. The inner edges of the posts must be 3.66 metres (4 yards) apart, and the lower edge of the crossbar must be 2.14 metres (7 feet) above the ground. Each team plays with 11 players on the field during the game including the goalie.*

Players positions that are included in the game are :

- *Forward: As shown by players A, B, C and D.*
- *Midfielders: As shown by players E, F and G.*
- *Fullbacks: As shown by players H, I and J.*
- *Goalie: As shown by player K*

Using the picture of a hockey field below, answer the questions that follow:



(A) **The coordinates of the centroid of ΔEHJ are:**

(a) $\left(\frac{-2}{3}, 1\right)$ (b) $\left(1, \frac{-2}{3}\right)$

(c) $\left(\frac{2}{3}, 1\right)$ (d) $\left(\frac{-2}{3}, -1\right)$

(B) **If a player P needs to be at equal distances from A and G, such that A, P and G are in straight line, then position of P will be given by:**

(a) $\left(\frac{-3}{2}, 2\right)$ (b) $\left(2, \frac{-3}{2}\right)$

(c) $\left(2, \frac{3}{2}\right)$ (d) (–2, –3)

(C) **The point on *x*-axis equidistant from I and E is:**

(a) $\left(\frac{1}{2}, 0\right)$ (b) $\left(0, \frac{-1}{2}\right)$

(c) $\left(\frac{-1}{2}, 0\right)$ (d) $\left(0, \frac{1}{2}\right)$

(D) **What are the coordinates of the position of a player Q such that his distance from K is twice his distance from E and K, Q and E are collinear?**

(a) (1, 0) (b) (0, 1)
(c) (–2, 1) (d) (–1, 0)

(E) **The point on y-axis equidistant from B and C is:**

(a) (–1, 0) (b) (0, –1)
(c) (1, 0) (d) (0, 1)

[CBSE Term-1 SQP 2021]

Ans. (C) *(a)* $\left(\frac{1}{2}, 0\right)$

Let the point on x-axis equidistant from I(-1, 1) and E(2, 1) be (x, 0) then

$$\sqrt{(x+1)^2+(0-1)^2} = \sqrt{(x-2)^2+(0-1)^2}$$

$x^2 + 1 + 2x + 1 = x^2 + 4 - 4x + 1$

$6x = 3$, *So* $x = 1/2$

∴ The required point is (1/2, 0)

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: Let the required point on *x*-axis be X (*x*, 0).

∴ XI = EX

Since, I = (–1, 1), E = (2, 1)

∴ Using distance formula,

$\Rightarrow \sqrt{(-1-x)^2+(1-0)^2} = \sqrt{(x-2)^2+(0-1)^2}$

$\Rightarrow (-1-x)^2 + 1^2 = (x-2)^2 + (-1)^2$ (Squaring both sides)

$\Rightarrow 1 + 2x + x^2 + 1 = x^2 - 4x + 4 + 1$

$\Rightarrow 2x + 2 = -4x + 5$

$\Rightarrow 6x = 3$

$\Rightarrow x = \frac{3}{6} = \frac{1}{2}$

∴ Required point on *x*-axis = $\left(\frac{1}{2}, 0\right)$

(E) *(d) (0, 1)*

Let the point on *y*-axis equidistant from B(4, 3) and C(4, –1) be (0, *y*) then

$$\sqrt{(4-0)^2+(3-y)^2} = \sqrt{(4-0)^2+(y+1)^2}$$

$16 + y^2 + 9 - 6y = 16 + y^2 + 1 + 2y$

$-8y = -8$

So, $y = 1$

∴ the required point is (0, 1)

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: Let the required point on *y*-axis be Y(0, *y*).

Then, according to question,

BY = CY

From the graph, B = (4, 3) and C = (4, –1)

∴ Using distance formula,

$\sqrt{(4-0)^2+(3-y)^2} = \sqrt{(0-4)^2+(y+1)^2}$

$\Rightarrow (-4)^2 + (3-y)^2 = (-4)^2 + (y+1)^2$ [Squaring both sides]

$\Rightarrow 16 + 9 - 6y + y^2 = 16 + y^2 + 2y + 1$

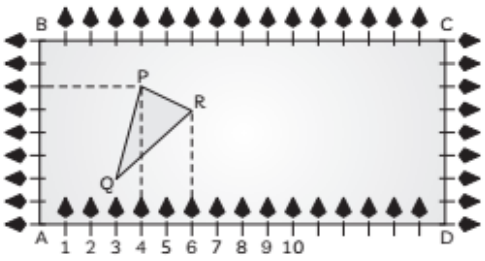
$\Rightarrow -6y + 25 = 2y + 17$

$\Rightarrow -8y = -8$

$\Rightarrow y = 1$

∴ Required point on *y*-axis = (0, 1)

51. *Class X students of a secondary school in Krishanagar have been allotted a rectangular plot of a land for gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.*



(A) Considering A as the origin, what are the coordinates of A?

(a) (0, 1) **(b) (1, 0)**
(c) (0, 0) **(d) (–1, –1)**

(B) What are the coordinates of P?

(a) (4, 6) **(b) (6, 4)**
(c) (4, 5) **(d) (5, 4)**

(C) What are the coordinates of R?

(a) (6, 5) **(b) (5, 6)**
(c) (6, 0) **(d) (7, 4)**

(D) What are the coordinates of D?

(a) (16, 0) **(b) (0, 0)**
(c) (0, 16) **(d) (16, 1)**

(E) What are the coordinates of P, if D is taken as the origin?

(a) (12, 2) **(b) (–12, 6)**
(c) (12, 3) **(d) (6, 10)**

[CBSE SQP 2020]

Ans. (A) *(c) (0, 0)*

Explanation: the coordinates of A are (0, 0). Given, that A is the origin.

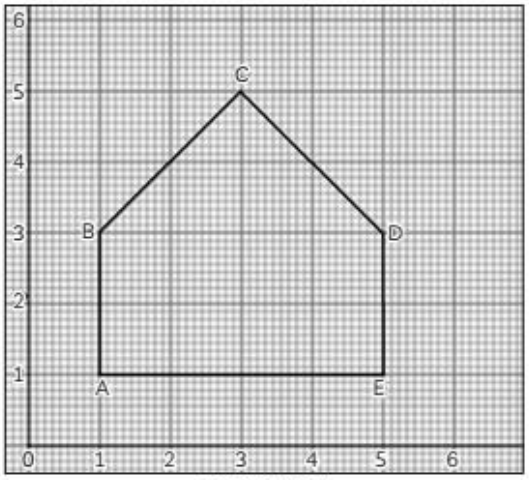
(B) (a) *(4, 6)*

Explanation: The coordinates of P are (4, 6).

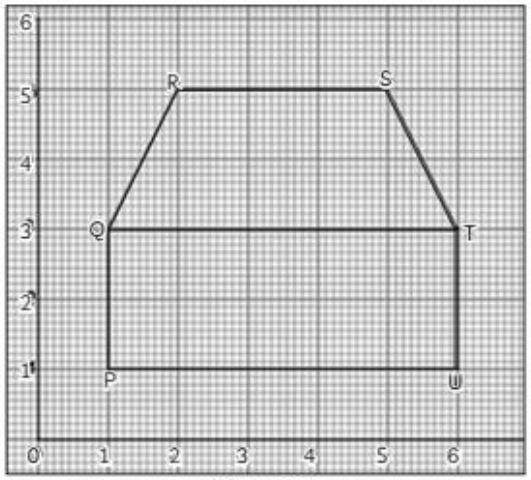
(C) (a) *(6, 5)*

Explanation: The coordinates of R are (6, 5).

52. *The blowing diagrams show the front view and side view of a hut.*



FRONT VIEW



SIDE VIEW

(A) Refer to the front view.
The coordinates of the point C are:

(a) (5, 3) (b) (- 3, 5)

(c) (3, 5) (d) (3, 6)

(B) Refer to the side view:
The mid-point of the segment joining Q and R is:

(a) $\left(0, \frac{3}{2}\right)$ (b) $\left(\frac{1}{2}, 0\right)$

(c) $\left(0, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, 4\right)$

(C) Refer to front view.
The distance between the points A and D is:

(a) $\sqrt{20}$ units (b) $\sqrt{24}$ units

(c) 20 units (d) $\sqrt{52}$ units

(D) Refer to side view.
The coordinates of the point that divides the line segment joining the points P and S in the ratio 1 : 3 is:

(a) (1, 3) (b) (2, 3)

(c) (2, 2) (d) (–2, –2)

(E) Refer to side view.
If a point (x, y) is equidistant from the points Q and S, then:

(a) $2x - y = 10$ (b) $2x + y = 10$

(c) $2x - y = -10$ (d) $2x + y = -10$

Ans. (A) *(c) (3, 5)*

Explanation: The coordinates (x, y) of any point are the distances of the point from the y-axis and x-axis respectively. Therefore, coordinates of point C are (3, 5).

(C) *(a) $\sqrt{20}$ units*

Explanation: The coordinates of points A and D are (1, 1) and (5, 3) respectively. Applying the distance formula, we get

$AD = \sqrt{(5-1)^2 + (3-1)^2} = \sqrt{4^2 + 2^2}$

$= \sqrt{16+4} = \sqrt{20}$ units

(E) *(b) $2x + y = 10$*

Explanation: The coordinates of the points Q and S are (1, 3) and (5, 5) respectively.

If a point (x, y) is equidistant from Q and S, then, the distances of the point (x, y) from Q and S will be equal.

Applying distance formula, we get

$$\sqrt{(x-1)^2 + (y-3)^2} = \sqrt{(x-5)^2 + (y-5)^2}$$

Squaring both sides,

$(x-1)^2 + (y-3)^2 = (x-5)^2 + (y-5)^2$

Simplifying, we get

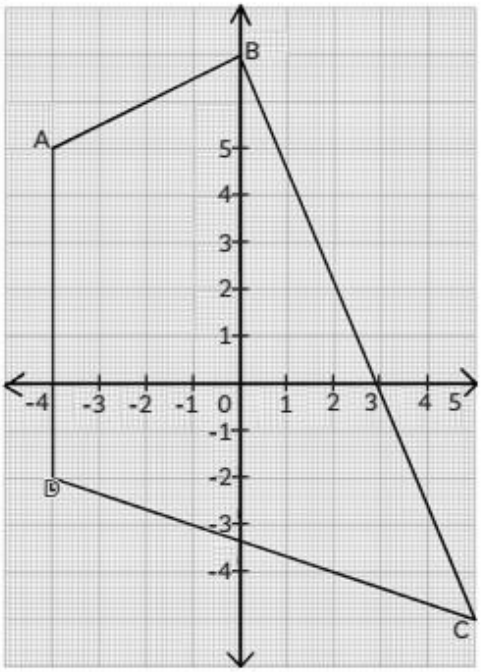
$x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 - 10x + 25 + y^2 - 10y + 25$

$\Rightarrow -2x - 6y + 10 = -10x - 10y + 50$

$\Rightarrow 8x + 4y = 40$

$\Rightarrow 2x + y = 10$

53. *A farmer has a plot of land in the shape of a quadrilateral as shown below:*



(A) Find the image of the vertex A on the *y*-axis.

(B) Find the image of the D on the *x*-axis.

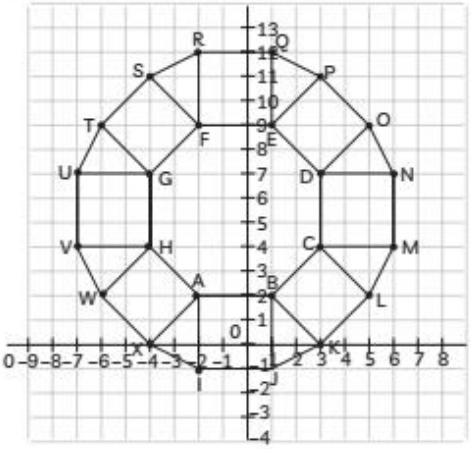
(C) Find the distances BC and CD.

Ans. (A) The image of any point on the *y*-axis will have the same *y*-coordinate, but its *x*-coordinate will be negative of its earlier value. As coordinates of A are (–4, 5), so coordinates of its image on *y*-axis will be (4, 5).

(B) The image of any point on the *x*-axis will have the same *x*-coordinate, but its *y*-coordinate will be negative of its earlier value. As coordinates of D are (–4, –2), so coordinates of its image on *x*-axis will be (–4, 2).

54. *A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons.*

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern



Use the above figure to answer the questions that follow:

(A) What is the length of the line segment joining points B and F?

(B) The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z?

(C) What are the coordinates of the point on y axis equidistant from A and G?

OR

What is the area of trapezium AFGH?

[CBSE SQP Std. 2022]

Ans. (A) B(1,2), F(-2,9)

$BF^2 = (-2 - 1)^2 + (9 - 2)^2$

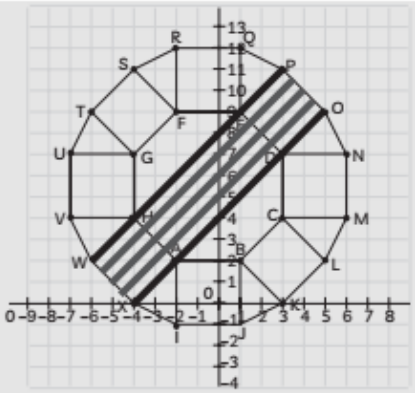
$= (-3)^2 + (7)^2$

$= 9 + 49$

$= 58$

So, $BF = \sqrt{58}$ units

(B)



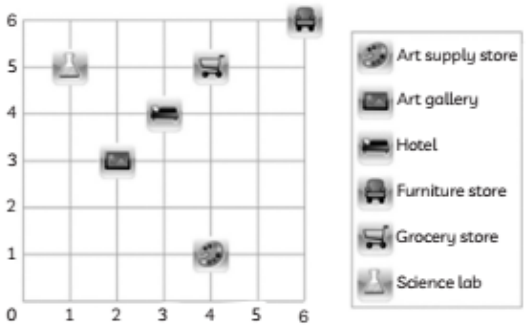
W(−6, 2), X(−4, 0), O(5, 9), P(3, 11)
Clearly WXOP is a rectangle
Point of intersection of diagonals of a rectangle is the mid point of the diagonals.
So the required point is mid point of WO or XP

$$= \left(\frac{-6+5}{2}, \frac{2+9}{2}\right)$$

$$= \left(\frac{-1}{2}, \frac{11}{2}\right)$$

[CBSE Marking Scheme SQP Std. 2022]

55. *A rough coordinate map of Sumit's locality is shown below:*



(A) What are the coordinates of the science lab?

(B) What is the distance between furniture store and art supply store?

(C) Sumit first goes to the grocery store from the science lab and then to the hotel. Find the total distance travelled by him.

Ans. (A) The x-coordinate and y-coordinate of any point in the cartesian plane are the distances of the point from the y-axis and x-axis respectively.

As the distance of the science lab from y-axis is 1 unit and from x-axis is 5 units, its coordinates are (1, 5).

(B) The coordinates of furniture store and art supply store are (6, 6) and (4, 1) respectively.

Distance between both the stores

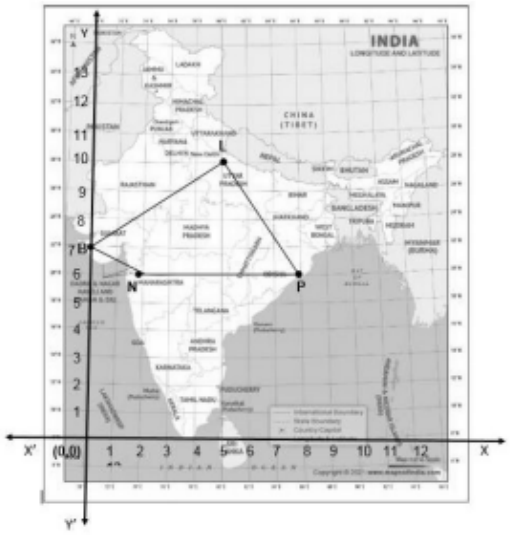
$$= \sqrt{[(4-6)^2 + (1-6)^2]}$$

$$= \sqrt{(-2)^2 + (-5)^2}$$

$$= \sqrt{4+25}$$

$$= \sqrt{29} \text{ units}$$

56. *In a GPS, The lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the given figure below.*



Based on the above information answer the following questions using the coordinate geometry.

(A) Find the distance between Lucknow (L) to Bhuj (B).

(B) If Kota (K), internally divide the line segment joining Lucknow (L) to Bhuj (B) into 3 : 2 then find the coordinate of Kota (K).

(C) Name the type of triangle formed by the places Lucknow (L), Nashik (N) and Puri (P)

OR

Find a place (point) on the longitude (y-axis) which is equidistant from the points Lucknow (L) and Puri (P).

[CBSE SQP Basic 2022]

Ans. (A) $LB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Rightarrow LB = \sqrt{(0-5)^2+(7-10)^2}$

$LB = \sqrt{(5)^2+(3)^2}$

$\Rightarrow LB = \sqrt{25+9}$

$LB = \sqrt{34}$

Hence, the distance is $150\sqrt{34}$ km

(B) Coordinate of Kota (K) is

$$\left(\frac{3\times5+2\times0}{3+2}, \frac{3\times7+2\times10}{3+2}\right)$$

$$=\left(\frac{15+0}{5}, \frac{21+20}{5}\right)=\left(3, \frac{41}{5}\right)$$

[CBSE Marking Scheme SQP Basic 2022]

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

57. Find the value of k, if P(4, –2) is the mid-point of the line segment joining the points A($5k$, 3) and B($-k$, –7). [CBSE 2010]

Ans. Since, P(4, –2) is the mid-point of the line segment joining the points A($5k$, 3) and B($-k$, –7)

∴ By mid-point formula,

$$\therefore \quad P(4,-2)=\left(\frac{5k-k}{2}, \frac{3-7}{2}\right)$$

$$\frac{4k}{2}=4$$

$$\Rightarrow \quad k=2$$

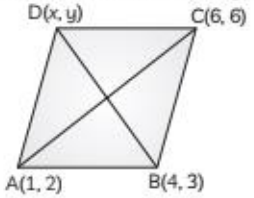
Hence, the value of k is 2.

58. What is the distance between the points A(c, 0) and B(0, –c)? [CBSE 2011]

59. The Dockland Building at the port of Hamburg is an astoundingly perfect parallelogram and an architectural marvel!

If A(1, 2), B(4, 3) and C(6, 6) are the three vertices of the building in the shape of the parallelogram ABCD then find the coordinates of the fourth vertex D. [CBSE 2010]

Ans. Let, the coordinates of D be (x, y).



Since, ABCD is a parallelogram, so diagonals AC and BD bisect each other.

∴ Mid-point of BD = Mid-point of AC

$$\left(\frac{4+x}{2}, \frac{3+y}{2}\right)=\left(\frac{1+6}{2}, \frac{2+6}{2}\right)$$

$\Rightarrow \quad 4+x=7$

and $\quad 3+y=8$

$\Rightarrow \quad x=3$

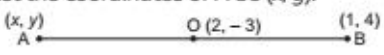
and $\quad y=5$

Hence, the coordinates of D are (3, 5).

60. We all know that oranges are a very rich source of vitamin-C and it has a near circular cross section when cut into two pieces, vertically. From the point of view of coordinate geometry, we can easily find the coordinates of the centre, if the coordinates of the ends of the diameter are known and vice versa.

Find the coordinates of a point A, where AB is the diameter of an orange cut into two parts, whose centre is (2, –3) and B(1, 4). [Mod. Diksha]

Ans. Let the coordinates of A be (x, y).



We know, centre of a circle is mid-point of its diameter.

$$\therefore \quad (2,-3)=\left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

$$\frac{x+1}{2}=2$$

$\Rightarrow \quad x = 4 - 1 = 3$

and $\quad \frac{y+4}{2} = -3$

$y = -6 - 4 = -10$

$\therefore$ Coordinates of A are (3, –10).

61. These days we don't feel lost when we go to an unknown place. Thanks to Google Maps! Do you know that you can easily find the distance between any two places if their coordinates are known to you by using the distance formula?

Find the distance between the two points (a, b) and (–a, –b) using distance formula.

[Mod. CBSE 2019]

Ans. Given: points are A (a, b) and B (– a, – b).

By the distance formula:

Required distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here, $\quad x_1 = a, y_1 = b$

$x_2 = -a, y_2 = -b$

$\therefore \quad AB = \sqrt{(-a - a)^2 + (-b - b)^2}$

$= \sqrt{(-2a)^2 + (-2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2 + b^2}$ units

Hence, the distance between the given points is $2\sqrt{a^2 + b^2}$ units.

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

62. Find the coordinates of the point which divides the line segment joining the points (4, –3) and (8, 5) in the ratio 3 : 1 internally.

[CBSE SQP 2020]

Ans. Let P (x, y) be the required point. Using section formula

$$\left\{\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right\} = (x, y)$$

$$x = \frac{3(8) + 1(4)}{3 + 1}$$

$$y = \frac{3(5) + 1(-3)}{3 + 1}$$

$x = 7, y = 3$

(7, 3) is the required point

[CBSE Marking Scheme SQP 2020]

63. Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5). **[CBSE SQP 2020]**

Ans. Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5)

Given, $\quad AP = BP$. So, $AP^2 = BP^2$

$(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$x^2 - 14x + 49 + y^2 - 2y + 1$

$= x^2 - 6x + 9 + y^2 - 10y + 25$

$x - y = 2$

[CBSE Marking Scheme SQP 2020]

64. Find the point on x-axis which is equidistant from the points (2, –2) and (–4, 2).

[CBSE SQP 2020]

65. P(–2, 5) and Q(3, 2) are two points. Find the co-ordinates of the point R on PQ such that PR = 2QR. **[CBSE SQP 2020]**

Ans. PR : QR = 2 : 1

$$R\left(\frac{2(3) + 1(-2)}{2 + 1}, \frac{2(2) + 1(5)}{2 + 1}\right)$$

$$R\left(\frac{4}{3}, 3\right)$$

[CBSE Marking Scheme SQP 2020]

Explanation: Let, the coordinates of R be (x, y).

Given, P(–2, 5) and Q(3, 2) are two points

and $PR = 2QR$

or, $\frac{PR}{QR} = \frac{2}{1}$

So, point R divides line PQ in the ratio of 2 : 1.

$$\underset{P}{(-2, 5)} \quad \overset{2}{\text{———}} \quad \underset{(x, y)}{R} \quad \overset{1}{\text{———}} \quad \underset{Q}{(3, 2)}$$

By section formula,

$$R(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Here, $m = 2, \quad n = 1$

$x_1 = -2, \quad y_1 = 5$

$x_2 = 3, \quad y_2 = 2$

$$\therefore \quad R(x, y) = \left(\frac{2\times3+1\times(-2)}{2+1}, \frac{2\times2+1\times5}{2+1}\right)$$

$$= \left(\frac{6-2}{3}, \frac{4+5}{3}\right)$$

$$= \left(\frac{4}{3}, 3\right)$$

Hence, the coordinates of point R are $\left(\frac{4}{3}, 3\right)$.

66. The mid-point of the line segment joining A(2a, 4) and B(–2, 3b) is (1, 2a + 1). Find the values of a and b. [CBSE 2019]

Ans. Let, P be the mid-point of the line AB. Coordinates of A are (2a, 4); B are (–2, 3b) and P are (1, 2a + 1)

By the mid-point formula:

Coordinates of the mid-point of AB

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Here, $x_1 = 2a \quad y_1 = 4$

$x_2 = -2 \quad y_2 = 3b$

$x = 1 \quad y = 2a + 1$

$$\therefore \quad 1 = \frac{2a + (-2)}{2} \text{ and } 2a + 1 = \frac{4 + 3b}{2}$$

$\Rightarrow \quad 2a = 4$ and $4a + 2 = 4 + 3b$

$\Rightarrow \quad a = 2$ and $4a - 3b = 2$

Put $a = 2$ in $4a - 3b = 2$, we get

$$4(2) - 3b = 2$$

$$\Rightarrow \quad -3b = 2 - 8 = -6$$

$$\Rightarrow \quad b = 2$$

Hence, the values of a is 2 and b is 2.

67. Determine the ratio in which the line $y - x + 2 = 0$ divides the line segment joining the points (3, –1) and (8, 9). [Diksha]

Ans. Let, line $y - x + 2 = 0$ divides the points (3, –1) and (8, 9) in the ratio $k : 1$ at point P.

$\therefore$ Co-ordinates of the point P

$$= \left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$$

Also, point P lies on line $y - x + 2 = 0$, so, it must satisfy this equation.

$$\therefore \quad \left(\frac{9k-1}{k+1}\right) - \left(\frac{8k+3}{k+1}\right) + 2 = 0$$

$$\Rightarrow \quad 9k - 1 - 8k - 3 + 2k + 2 = 0$$

$$\Rightarrow \quad 3k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{3}$$

Hence, the required ratio is 2 : 3.

68. If two adjacent vertices of a parallelogram are (3, 2) and (–1, 0) and the diagonals intersect at (2, –5), then find the coordinates of the other two vertices. [CBSE 2017]

69. In what ratio does the point P(–4, 6) divide the line segment joining the points A(–6, 10) and B(3, –8)? [CBSE 2017]

Ans. Let, the ratio in which P divides line AB be $k : 1$.

$$\underset{A(-6, 10)}{\bullet} \quad \overset{P(-4, 6)}{\bullet} \quad \underset{B(3, -8)}{\bullet}$$

By section formula,

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Here, $x = -4, \quad y = 6$

$x_1 = -6, \quad y_1 = 10$

$x_2 = 3, \quad y_2 = -8$

$m = k, \quad n = 1$

Now, $P(-4, 6) = \left[\frac{k\times3+1\times(-6)}{k+1}, \frac{k \times (-8) + (1) \times 10}{k + 1}\right]$

$$\therefore \quad -4 = \frac{3k-6}{k+1} \quad \text{and} \quad 6 = \frac{-8k+10}{k+1}$$

$$\Rightarrow \quad -4k - 4 = 3k - 6, \quad \Rightarrow \quad 6k + 6 = -8k + 10$$

$$\Rightarrow \quad -7k = -2, \quad \Rightarrow \quad 14k = 4$$

$$\Rightarrow \quad k = \frac{2}{7}, \quad \Rightarrow \quad k = \frac{2}{7}$$

Hence, the required ratio is 2 : 7.

70. Find the ratio in which the point (–3, k) divides the line-segment joining the points (–5, –4) and (–2, 3). Also, find the value of k. [CBSE 2016]

71. The coordinates of houses of Sonu and Labhoo are (7, 3) and (4, 3) respectively. Coordinates of their school are (2, 2). If both leave their house at the same time in the morning and also reach school in same time, then who travel faster? [Diksha]

Ans. Distance between Sonu's house and school

$$= \sqrt{(2-7)^2+(2-3)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right]$$

$$= \sqrt{25+1} = \sqrt{26}$$

Distance between Labhoo's house and school

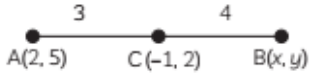
$$= \sqrt{(2-4)^2+(2-3)^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

So, distance of Sonu's house from school is more. Therefore, Sonu travels faster.

72. If the point C(–1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B. [CBSE 2013]

Ans. Since, C divides AB in the ratio 3 : 4, we have



By section formula,

$$C(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{3x+8}{7}, \frac{3y+20}{7}\right)$$

i.e., $C\left(\frac{3x+8}{7}, \frac{3y+20}{7}\right) = C(-1, 2)$

$$\Rightarrow \quad \frac{3x+8}{7} = -1$$

and $\quad \frac{3y+20}{7} = 2$

$$\Rightarrow \quad x = -5 \text{ and } y = -2$$

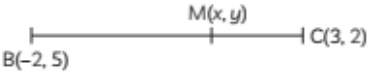
Thus, coordinates of B are (–5, –2).

73. Find the coordinates of the point M on the line joining B(–2, 5) and C(3, 2) such that BM = 2CM. [Delhi Gov. SQP 2022]

Ans. Given, BC is a line segment with coordinates of B as (–2, 5) and coordinates of C as (3, 2)

and $\quad BM = 2\ CM$

$$\frac{BM}{CM} = \frac{2}{1}$$



Let the coordinates of M be (x, y).

So, by the section formula,

$$M(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Here, $\quad m = 2, \quad n = 1$

$x_1 = -2, \quad y_1 = 5$

$x_2 = 3, \quad y_2 = 2$

$$\therefore \quad M(x, y) = \left(\frac{2\times3+1\times(-2)}{2+1}, \frac{2\times2+1\times5}{2+1}\right)$$

$$= \left(\frac{6-2}{3}, \frac{4+5}{3}\right)$$

$$= \frac{4}{3}, 3$$

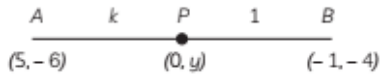
$$\therefore \quad M(x, y) = \left(\frac{4}{3}, 3\right)$$

SHORT ANSWER Type Questions (SA-II)

[3 marks]

74. Find the ratio in which y-axis divides the line segment joining the points A(5, –6) and B(–1, –4). Also, find the coordinates of the point of division.

Ans. Given, points are: A(5, –6) and B(–1, –4).



Let, point P(0, y) be the point that divides the given line-segment. Since, point P is on y-axis, its x-coordinate is zero.

Let the ratio in which P divides AB be k : 1.

By the section formula,

$$P(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

Here, $\quad m_1 = k, \quad m_2 = 1$

$x_1 = 5, \quad y_1 = -6$

$x_2 = -1, \quad y_2 = -4$

$x = 0, \quad y = y$

$$(0, y) = \left(\frac{k\times(-1)+1\times5}{k+1}, \frac{k\times(-4)+1\times(-6)}{k+1}\right)$$

On comparing the x-coordinate both sides, we get

$$0 = \frac{-k+5}{k+1}$$

$$\Rightarrow \quad k = 5$$

Hence, the required ratio is 5 : 1.

Now, comparing the y-coordinate, we get

$$y = \frac{k \times (-4) + 1 \times (-6)}{k + 1}$$

$$= \frac{5\times(-4)-6}{5+1} \qquad [\because\ k = 5]$$

$$= \frac{-20-6}{6} = \frac{-26}{6} = \frac{-13}{3}$$

Hence, the coordinates of point P are $\left(0, \frac{-13}{3}\right)$.

75. The line segment joining the points A $(3a - 2, 2 + a)$ and $(4 - 3a, a - 1)$ is trisected by the points P and Q. If P lies on the line $2x - 3y + 5 = 0$, find a. [British Council 2022]

76. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, –5) and R(–3, 6), find the coordinates of P. [CBSE 2010]

77. If the distance between the points $(4, k)$ and $(1, 0)$ is 5, what can be the possible values of k? [CBSE 2017]

Ans. Given, points are A$(4, k)$ and B$(1, 0)$.

Distance between points A and B = 5 units

According to the distance formula,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 4, y_1 = k$

$x_2 = 1, y_2 = 0$

$$\therefore \quad 5 = \sqrt{(1-4)^2 + (0-k)^2}$$

$$\Rightarrow \quad (5)^2 = (-3)^2 + (-k)^2$$

[On squaring both sides]

$$\Rightarrow \quad 25 = 9 + k^2$$

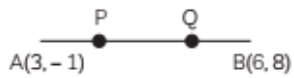
$$\Rightarrow \quad k^2 = 25 - 9 = 16$$

$$\Rightarrow \quad k = \pm\sqrt{16} = \pm 4$$

Hence, the possible values of k are 4 and –4.

78. Find the co-ordinates of the points of trisection of the line segment joining the points (3, –1) and (6, 8). [CBSE 2011]

Ans. Let the given points be A and B and P and Q be the points of trisection of AB, as shown in the figure.



Here, P divides AB is the ratio 1 : 2 and Q divides AB in the ratio 2 : 1.

By section formula

$$P = \left(\frac{1\times6+2\times3}{1+2}, \frac{1\times8+2\times(-1)}{1+2}\right)$$

and $Q = \left(\frac{2\times6+1\times3}{2+1}, \frac{2\times8+1\times(-1)}{2+1}\right)$

$$\Rightarrow \quad P = \left(\frac{6+6}{3}, \frac{8-2}{3}\right) \text{and } Q = \left(\frac{12+3}{3}, \frac{16-1}{3}\right)$$

i.e., P(4, 2) and Q(5, 5).

79. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there? [NCERT]

Ans. We know that, any point on x-axis is of the form $(x, 0)$.

Let P $(x, 0)$ be the point on x-axis having $2\sqrt{5}$ unit from the point Q (7, –4).

Distance between P $(x, 0)$ and Q (7, –4) using distance formula is,

$$PQ = \sqrt{(7-x)^2 + (-4-0)^2}$$

$$= \sqrt{(7-x)^2 + (-4)^2}$$

According to given condition,

$$PQ = 2\sqrt{5}$$

$$\Rightarrow \quad (PQ)^2 = \left(2\sqrt{5}\right)^2$$

$$\Rightarrow \quad (7-x)^2 + (-4)^2 = \left(2\sqrt{5}\right)^2$$

$$\Rightarrow \quad 49 + x^2 - 14x + 16 = 20$$

$$\Rightarrow \quad x^2 - 14x + 45 = 0$$

$$\Rightarrow \quad x^2 - 9x - 5x + 45 = 0$$

[using factorisation method]

$$\Rightarrow \quad x(x-9) - 5(x-9) = 0$$

$$\Rightarrow \quad (x-9)(x-5) = 0$$

$$\Rightarrow \quad x = 9, 5.$$

Hence, there are two points that lie on x-axis, namely, (5, 0) and (9, 0), having a distance of $2\sqrt{5}$ units from the point (7, –4).

80. What type of a quadrilateral do the points A(2, –2), B(7, 3), C(11, –1) and D(6, –6), taken in that order, form? [NCERT]

Ans. To find the type of quadrilateral, we will find the length of all four sides and the length of diagonals.

We know that distance between points (x_1, y_1) and (x_2, y_2) is given by,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, A(2, –2), B(7, 3), C(11, –1) and D(6, –6).

$$\therefore \ AB = \sqrt{(7-2)^2 + (3+2)^2} = \sqrt{5^2 + 5^2}$$

$$= \sqrt{25+25} = 5\sqrt{2}$$

$$BC = \sqrt{(11-7)^2+(-1-3)^2} = \sqrt{4^2+(-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$CD = \sqrt{(6-11)^2+(-6+1)^2} = \sqrt{5^2+(-5)^2}$$
$$= \sqrt{25+25} = 5\sqrt{2}$$

$$AD = \sqrt{(6-2)^2+(-6+2)^2} = \sqrt{4^2+(-4)^2}$$
$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Also,

$$AC = \sqrt{(11-2)^2+(-1+2)^2} = \sqrt{9^2+1^2}$$
$$= \sqrt{81+1} = \sqrt{82}$$

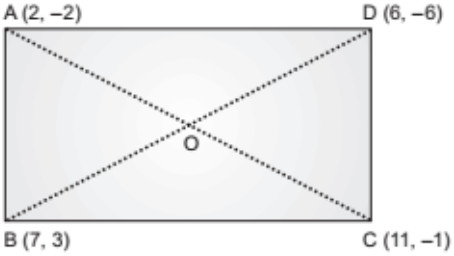
And,

$$BD = \sqrt{(6-7)^2+(-6-3)^2} = \sqrt{(-1)^2+(-9)^2}$$
$$= \sqrt{1+81} = \sqrt{82}$$

Here, we see that length of opposite sides are equal.

i.e. $AB = DC = 5\sqrt{2}$

$AD = BC = 4\sqrt{2}$



Also, length of diagonals are equal.

i.e., $AC = BD = \sqrt{82}$

This shows that the given points form a rectangle.

Concept Applied

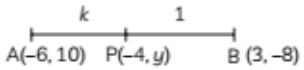
- *Step 1. Determine AB, BC, CD, AD, BD and AC using distance formula.*
- *Step 2. Check the following conditions to determine type of quadrilateral:*
  - *(1) If all sides and diagonals are equal then it is a square.*
  - *(2) If all sides are equal but diagonals are not equal, then it is a rhombus.*
  - *(3) If opposite sides and diagonals are also equal, then it is a rectangle.*

81. **Point P divides the line segment joining the points A(2, 1) and B(5, –8) such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k.**

82. **Find a point which is equidistant from the points A(–5, 4) and B(–1, 6)? How many such points are there?** **[NCERT Exemplar]**

83. **In what ratio does the point P(–4, y) divide the line segment joining the points A(–6, 10) and B(3, –8)? Also, find the value of y.** **[CBSE 2020]**

Ans. Let the point P divides the line segment AB in the ratio of $k : 1$.



By the section formula, the coordinates of P are:

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Here, $m = k, n = 1, x_1 = -6, y_1 = 10,$
$x_2 = 3, y_2 = -8$

and $x = -4, y = y$

$$\therefore \quad P(-4, y) = \left(\frac{3k-6}{k+1}, \frac{-8k+10}{k+1}\right)$$

On comparing 'x' and 'y' coordinates, we have

$$-4 = \frac{3k-6}{k+1}$$
$$\Rightarrow \quad -4k - 4 = 3k - 6$$
$$\Rightarrow \quad -7k = -2$$
$$\Rightarrow \quad k = \frac{2}{7}$$

$\therefore$ The ratio is $\frac{2}{7}$.

and $$y = \frac{-8k+10}{k+1}$$

Put the value of 'k', we get

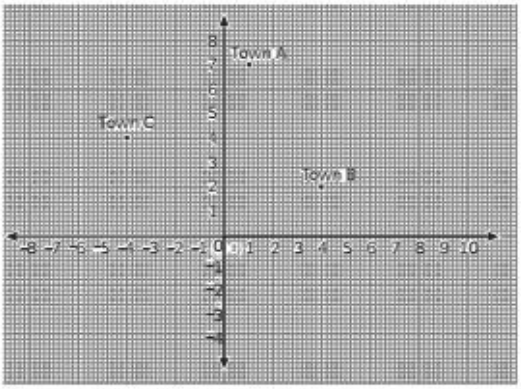
$$y = \frac{-8 \times \frac{2}{7} + 10}{\frac{2}{7} + 1}$$
$$= \frac{-16+70}{2+7} = \frac{54}{9}$$
$$= 6$$

Hence, the value of 'y' is 6.

84. **Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points (–2, –5) and (6, 3). Find the coordinates of the point of intersection.** **[Delhi Gov. QB 2022, CBSE 2019]**

85. **Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometown represented by Town A and Town B respectively. Town A and Town B are connected by trains from the same station**

C (in the given figure) in Delhi. Based on the given situation, answer the following questions:



(A) Who will travel more distance, Seema or Aditya, to reach to their hometown?

(B) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point D.

Ans. (A) From the graph, we have A(1, 7), B(4, 2) C(–4, 4).

Distance travelled by Seema

$= AC$

$= \sqrt{(-4-1)^2+(4-7)^2}$

$= \sqrt{(-5)^2+(-3)^2} = \sqrt{25+9} = \sqrt{34}$

Distance travelled by Aditya

$= BC$

$= \sqrt{(-4-4)^2+(4-2)^2}$

$= \sqrt{(-8)^2+(2)^2}$

$= \sqrt{64+4} = \sqrt{68}$

As, BC > AC

∴ Aditya travels more distance.

(B) Coordinates of D = Mid-point of AB

$= \left(\frac{1+4}{2}, \frac{7+2}{2}\right)$

$= \left(\frac{5}{2}, \frac{9}{2}\right)$

86. Find the point on *y*-axis which is equidistant from the points (5, – 2) and (– 3, 2). **[CBSE 2016]**

Ans. Let the required point on *y*-axis be P(0, *b*).

Given, points are A(5, –2) and B(–3, 2).

Since, the points A and B are equidistant from point P.

∴ Distance AP = Distance BP

Applying the distance formula, we get

$$\sqrt{(x_1-x)^2+(y_1-y)^2} = \sqrt{(x_2-x)^2+(y_2-y)^2}$$

Here, $x = 0, \quad y = b$

$x_1 = 5, \quad y_1 = -2$

$x_2 = -3, \quad y_2 = 2$

$$\therefore \sqrt{(5-0)^2+(-2-b)^2} = \sqrt{(-3-0)^2+(2-b)^2}$$

On squaring both sides, we get

$\Rightarrow \quad 25 + (-2-b)^2 = 9 + (2-b)^2$

$\Rightarrow \quad 25 + 4 + b^2 + 4b = 9 + 4 + b^2 - 4b$

$\Rightarrow \quad 8b = -16$

$\Rightarrow \quad b = -2$

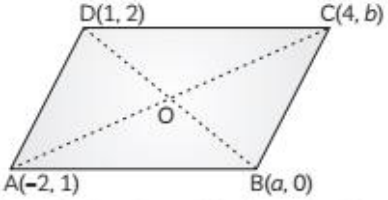
Hence, the required point is (0, – 2).

87. The line segment joining the points A(2, 1) and B(5, – 8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of *k*. **[CBSE 2013]**

88. If the point A (2, –4) is equidistant from P (3, 8) and Q (–10, *y*), find the values of *y*. Also find distance PQ. [CBSE 2014, 12, 11, 10]

89. If A(–2, 1), B(*a*, 0), C(4, *b*) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of *a* and *b*. Also, find the lengths of its sides.

Ans. Given, ABCD is a parallelogram, in which diagonals AC and BD bisect each other at O.



Now, O is the mid-point of AC and BD, respectively.

By the mid point formula:

Coordinates of O $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

For line BD, $O(x, y) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$

$= \left(\frac{a+1}{2}, 1\right)$ ...(i)

For line AC, $O(x, y) = \left(\frac{-2+4}{2}, \frac{1+b}{2}\right)$

$= \left(1, \frac{1+b}{2}\right)$ ...(ii)

From (i) and (ii), we get

$\frac{a+1}{2} = 1, \quad 1 = \frac{1+b}{2}$

$\Rightarrow \quad a = 1, \quad b = 1$

Length of side AB, using the distance formula:

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(a+2)^2 + (0-1)^2}$$

$$= \sqrt{(1+2)^2 + (0-1)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10} \text{ units}$$

Also, $BC = \sqrt{(4-a)^2 + (b-0)^2}$

$$BC = \sqrt{(4-1)^2 + (1-0)^2}$$

$$= \sqrt{9+1} = \sqrt{10} \text{ units}$$

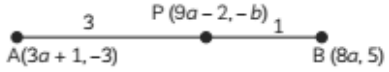
$\therefore$ $AB = CD = \sqrt{10}$ units

Hence, the length of all the four sides of parallelogram ABCD is $\sqrt{10}$ units.

90. Find the coordinates of the points of trisection of the line segment joining the points (3, –2) and (–3, –4). [CBSE 2017]

91. If P(9a – 2, –b) divides line segment joining A(3a + 1, –3) and B(8a, 5) in the ratio 3 : 1, find the values of a and b. [CBSE 2010]

Ans. It is given that P(9a – 2, –b) divides line segment joining A(3a + 1, –3) and B(8a, 5) in the ratio 3 : 1.



$\therefore$ By section formula,

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{3(8a) + 1(3a+1)}{3+1}, \frac{3(5) + 1(-3)}{3+1}\right)$$

$$\Rightarrow (9a - 2, -b) = \left(\frac{24a + 3a + 1}{4}, \frac{15 - 3}{4}\right)$$

$$\Rightarrow 9a - 2 = \frac{27a+1}{4} \text{ and } -b = \frac{15-3}{4}$$

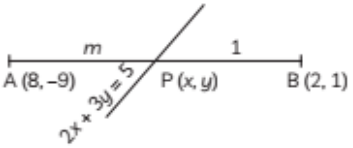
$$\Rightarrow 36a - 8 = 27a + 1 \text{ and } b = -\frac{12}{4}$$

$$\Rightarrow a = 1 \text{ and } b = -3$$

Hence, the required values of a and b are 1 and –3 respectively.

92. Find the ratio in which line 2x + 3y – 5 = 0 divides the line segment joining the points (8, –9) and (2, 1). Also, find the coordinates of the point of division. [CBSE 2013]

Ans. Let the line $2x + 3y - 5 = 0$ divide the line segment joining the points A(8, –9) and B(2, 1) in the ratio $m : 1$ at point P(x, y).



$\therefore$ Using section formula

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x, y) = \left(\frac{2m+8}{m+1}, \frac{m-9}{m+1}\right)$$

It is given that P lies on $2x + 3y - 5 = 0$.

$$\therefore 2\left(\frac{2m+8}{m+1}\right) + 3\left(\frac{m-9}{m+1}\right) - 5 = 0$$

$$\Rightarrow 2(2m + 8) + 3(m - 9) - 5(m + 1) = 0$$

$$\Rightarrow 4m + 16 + 3m - 27 - 5m - 5 = 0$$

$$\Rightarrow 2m - 16 = 0$$

$$\Rightarrow m = \frac{16}{2} = 8$$

$$\Rightarrow m : 1 = 8 : 1$$

$\therefore$ Coordinates of point $P = \left(\frac{2m+8}{m+1}, \frac{m-9}{m+1}\right)$

$$= \left(\frac{16+8}{9}, \frac{8-9}{9}\right)$$

$$= \left(\frac{24}{9}, \frac{-1}{9}\right)$$

$$= \left(\frac{8}{3}, \frac{-1}{9}\right)$$

Hence, the ratio is 8 : 1 and the point of division is $\left(\frac{8}{3}, \frac{-1}{9}\right)$.

93. Find the coordinates of a point on the x-axis which is equidistant from the points A(2, – 5) and B(–2, 9). [CBSE 2017]

Ans. Let, the coordinates of required point on x-axis be P(x, 0).

[y-coordinate will be zero, because point is on x-axis]

Point A(2, – 5) and point B(– 2, 9) are equidistant from point P.

$\therefore$ By distance formula

$$PA = PB$$

$$\sqrt{(x_1-x)^2+(y_1-y)^2} = \sqrt{(x_2-x)^2+(y_2-y)^2}$$

Here, $x_1 = 2, y_1 = -5, x_2 = -2, y_2 = 9$

$$\Rightarrow \sqrt{(2-x)^2+(-5-0)^2} = \sqrt{(-2-x)^2+(9-0)^2}$$

On squaring both sides, we get

$$4 + x^2 - 4x + 25 = 4 + x^2 + 4x + 81$$
$$\Rightarrow -8x = 81 - 25$$
$$\Rightarrow -8x = 56$$
$$\Rightarrow x = -7$$

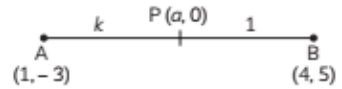
Hence, the coordinates of the required point on x-axis is $(-7, 0)$.

94. Write the coordinates of a point P on x-axis which is equidistant from the points A(–2, 0) and B(6, 0). [CBSE 2019]

95. The point R divides the line segment AB, where A(–4, 0) and B(0, 6) such that AR = $\frac{3}{4}$ AB. Find the coordinates of R. [CBSE 2019]

96. Find the ratio in which the line segment joining the points (1, –3) and (4, 5) is divided by x-axis? Also, find the coordinates of this point on x-axis. [CBSE 2019]

Ans. Let, the required point on x-axis be $(a, 0)$ and the required ratio be $k : 1$.



By the section formula, the co-ordinates of P are given by:

$$P(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Here, $m = k$, $n = 1$
$x_1 = 1$, $y_1 = -3$
$x_2 = 4$, $y_2 = 5$
$x = a$, $y = 0$

$$\therefore a = \frac{4k+1}{k+1}, \text{ and } 0 = \frac{5k-3}{k+1}$$
$$\Rightarrow 5k - 3 = 0$$
$$\Rightarrow k = \frac{3}{5}$$

Now, $$a = \frac{4\left(\frac{3}{5}\right)+1}{\frac{3}{5}+1}$$

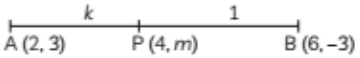
$$\Rightarrow a = \frac{\frac{12}{5}+1}{\frac{8}{5}}$$

$$\Rightarrow a = \frac{17}{8}$$

Hence, the ratio in which the line segment is divided, is 3 : 5 and the coordinates of the point of division are $\left(\frac{17}{8}, 0\right)$.

97. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, –3). Hence find m. [CBSE 2018]

Ans. Let the required ratio be $k : 1$.



$\therefore$ By the section formula

$$P(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Here $x_1 = 2, y_1 = 3$
$x_2 = 6, y_2 = -3$
$x = 4, y = m$
$m = k, n = 1$

$$\therefore P(4, m) = \left(\frac{k\times6+1\times2}{k+1}, \frac{k\times(-3)+1\times3}{k+1}\right)$$

$$\therefore 4 = \frac{6k+2}{k+1}$$
$$\Rightarrow 4k + 4 = 6k + 2$$
$$\Rightarrow 2k = 2$$
$$\Rightarrow k = 1$$

$\therefore$ Ratio in which P divides AB is 1 : 1.

Now $$m = \frac{-3k+3}{k+1}$$

$$\Rightarrow m = \frac{-3+3}{1+1} = 0$$

Hence, the value of m is 0.

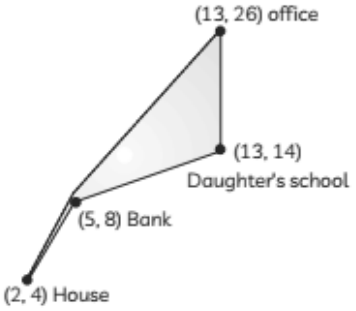
LONG ANSWER Type Questions (LA)

[4 & 5 marks]

98. Ayush starts walking from his house to his office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is extra distance travelled by Ayush in reaching his office (assume that all travelled distances covered are in straight lines)?

The house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13 ,26) and coordinates are in km. [NCERT Exemplar]

Ans. By given condition, we draw a figure in which every place is indicated with its coordinates.



∴ Using distance formula,

Distance between house and bank

$$= \sqrt{(5-2)^2+(8-4)^2}$$

$$= \sqrt{(3)^2+(4)^2} = \sqrt{25}$$

$$= 5 \text{ km}$$

Distance between bank and daughter's school

$$= \sqrt{(13-5)^2+(14-8)^2}$$

$$= \sqrt{8^2+6^2} = 10 \text{ km}$$

Distance between daughter's school and office

$$= \sqrt{(13-13)^2+(26-14)^2}$$

$$= \sqrt{0+12^2} = 12 \text{ km}$$

Total distance travelled = 5 + 10 + 12 = 27 km

Distance between house and office

$$= \sqrt{(13-2)^2+(26-4)^2}$$

$$= \sqrt{(11)^2+(22)^2}$$

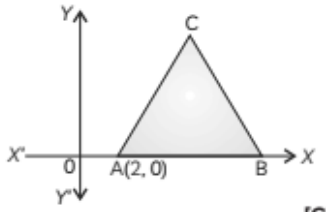
$$= \sqrt{121+484}$$

$$= \sqrt{605} = 24.59 \approx 24.6 \text{ km}$$

∴ Extra distance travelled = 27 – 24.6 = 2.4 km

Hence, Ayush travelled an extra distance of 2.4 km in reaching his office.

99. In the given figure, ΔABC is an equilateral triangle of side 3 units. Find the coordinates of the other two vertices.



[CBSE 2017]

Ans. Given an equilateral triangle ABC of side 3 units.

Also, coordinates of vertex A are (2, 0).

Let the coordinates of B = $(x, 0)$ and C = (x', y').

Then, using the distance formula,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$3 = \sqrt{(x-2)^2 + (0-0)^2}$$

On squaring both sides, we get

$$9 = (x-2)^2$$

$$\Rightarrow \quad x^2 + 4 - 4x = 9$$

$$\Rightarrow \quad x^2 - 4x - 5 = 0$$

$$\Rightarrow \quad x^2 - 5x + x - 5 = 0$$

$$\Rightarrow \quad (x-5)(x+1) = 0$$

$$\Rightarrow \quad x = 5, -1$$

But $x \neq -1$ (since it lies on positive x-axis)

∴ Coordinates of B are (5, 0).

Now, $AC = BC$

[∵ Sides of an equilateral triangle are equal]

or $AC^2 = BC^2$

By using the distance formula,

$$(x'-2)^2 + (y'-0)^2 = (x'-5)^2 + (y'-0)^2$$

$$\Rightarrow \quad x'^2 + 4 - 4x' + y'^2 = x'^2 + 25 - 10x' + y'^2$$

$$\Rightarrow \quad 6x' = 21$$

$$\Rightarrow \quad x' = \frac{7}{2}$$

Also, $AC = 3$ (given)

$$\sqrt{(x'-2)^2 + (y'-0)^2} = 3$$

$$\Rightarrow \quad x'^2 + 4 - 4x' + y'^2 = 9$$

[on squaring both sides]

$$\Rightarrow \quad \frac{49}{4} + 4 - 4 \times \frac{7}{2} + y'^2 = 9 \qquad \left[\because x' = \frac{7}{2}\right]$$

$$\Rightarrow \quad y'^2 = 9 - \frac{9}{4} = \frac{27}{4}$$

$$\Rightarrow \quad y' = \sqrt{\frac{27}{4}} = \pm\frac{3\sqrt{3}}{2}$$

But, C lies in the first quadrant.

$$\therefore \quad y' = \frac{3\sqrt{3}}{2}$$

∴ Coordinates of C are $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$.

Hence, the coordinates of B and C are (5, 0) and $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$, respectively.

100. Show that ΔABC, where A(–2, 0), B(2, 0), C(0, 2) and ΔPQR where P(–4, 0), Q(4, 0), R(0, 4) are similar triangles. [CBSE 2017]

Ans. Given: ΔABC with vertices A(–2, 0), B(2, 0), C(0, 2) and ΔPQR with vertices P(–4, 0) Q(4, 0) and R(0, 4).

In ΔABC, using distance formula,

$$AB = \sqrt{(2+2)^2+(0-0)^2}$$
$$= \sqrt{4^2 + 0^2}$$
$$= 4$$
$$BC = \sqrt{(0-2)^2+(2-0)^2}$$
$$= \sqrt{4 + 4} = \sqrt{8}$$
$$= 2\sqrt{2}$$
$$CA = \sqrt{(0+2)^2+(2-0)^2}$$
$$= \sqrt{4 + 4} = \sqrt{8}$$
$$= 2\sqrt{2}$$

Similarly, in ΔPQR, using distance formula,

$$\therefore \quad PQ = \sqrt{(4 + 4)^2 + (0 - 0)^2}$$
$$= \sqrt{8^2} = 8$$
$$QR = \sqrt{(0 - 4)^2 + (4 - 0)^2}$$
$$= \sqrt{16 + 16} = \sqrt{32}$$
$$= 4\sqrt{2}$$
$$PR = \sqrt{(0 + 4)^2 + (4 - 0)^2}$$
$$= \sqrt{16 + 16} = \sqrt{32}$$
$$= 4\sqrt{2}$$

Now, $\frac{AB}{PQ} = \frac{4}{8} = \frac{1}{2}$

$\frac{BC}{QR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$

and $\frac{CA}{PR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR}$

Since, the corresponding sides of ΔABC and ΔPQR are proportional,

$\therefore$ ΔABC ~ ΔPQR

Hence, proved.

101. If the point C(–1, 2) divides internally the line-segment joining the points A(2, 5) and B(x, y) in the ratio 3 : 4, find the value of $x^2 + y^2$. [CBSE 2016]

102. If (–4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle. [CBSE 2011, NCERT Exemplar]

103. Points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC.

(A) The median from A meets BC at D. Find the coordinates of point D. [CBSE 2010]

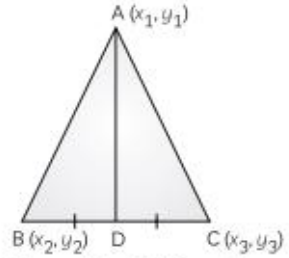
(B) Find the coordinates of the point P on AD such that AP : PD = 2 : 1. [CBSE 2010]

(C) Find the coordinates of points Q and R on medians BE and CF respectively, such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.

Ans. It is given that A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of ΔABC.

(A) We know that median bisects the opposite side into two equal parts *i.e.*, D is the mid-point of BC.

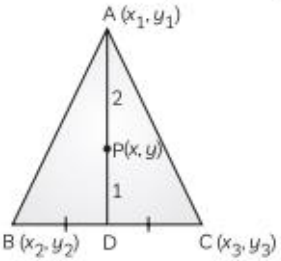
$\Rightarrow$ BD = DC



$\therefore$ Using mid-point formula,

$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

(B) Let the coordinates of point P be (x, y).



It is given that P (x, y) divides the line joining A (x_1, y_1) and D $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$ in the ratio 2 : 1.

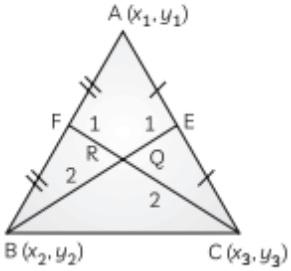
∴ Using section formula,

$$P(x, y) = \left(\frac{2\left(\frac{x_2+x_3}{2}\right)+1(x_1)}{2+1}, \frac{2\left(\frac{y_2+y_3}{2}\right)+1(y_1)}{2+1}\right)$$

$$= \left(\frac{x_2+x_3+x_1}{3}, \frac{y_2+y_3+y_1}{3}\right)$$

(C) It is given that Q and R are points on medians BE and CF, respectively such that

BQ : QE = 2 : 1 and CR : RF = 2 : 1



∵ BE is the median of AC.

⇒ E divides AC into two equal parts

∴ Mid-point of AC = Coordinates of E

$$\Rightarrow \quad E = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$$

So, using section formula,

Coordinates of point Q

$$= \left(\frac{2\left(\frac{x_1+x_3}{2}\right)+1(x_2)}{2+1}, \frac{2\left(\frac{y_1+y_3}{2}\right)+1(y_2)}{2+1}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_3+y_2}{3}\right)$$

∵ F is the mid-point of AB

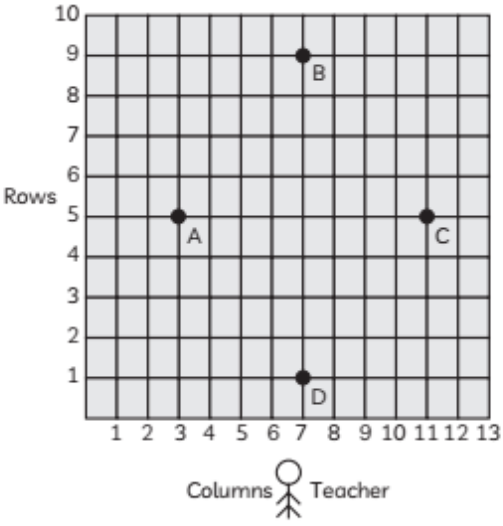
Coordinates of $F = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

∴ Coordinates of point R

$$= \left(\frac{2\left(\frac{x_1+x_2}{2}\right)+1(x_3)}{2+1}, \frac{2\left(\frac{y_1+y_2}{2}\right)+1(y_3)}{2+1}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

104. Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in the figure. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D? If so, what should be his position?



[NCERT Exemplar]